J/ψ and η_c in the nuclear medium: QCD sum rule approach*

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We investigate the masses of the lowest $c\bar{c}$ states, the J/ψ and η_c, in nuclear matter using QCD sum rules. Up to dimension four, the differences between the operator product expansions in vacuum and in medium arise from the density-dependent change in the gluon condensate and from a new contribution proportional to the nucleon expectation value of the twist-2 gluon operator. Both terms together give an attractive shift of about 5-10 MeV to the J/ψ and η_c masses in nuclear matter.

PACS: 11.55.Hx, 14.40.Gx
Keywords: QCD Sum rules; medium effects

Investigating the behaviour of heavy quark systems in a nuclear medium is of great interest, for several reasons. First, the ongoing discussion of J/ψ suppression in ultrarelativisitic heavy-ion collisions as a possible quark-gluon plasma signal requires detailed knowledge about the in-medium interactions of the J/ψ under "normal", non-plasma conditions. Furthermore, as Brodsky et al [1] pointed out, multigluon exchange can lead to an attractive potential between a $c\bar{c}$-meson and a nucleon, such that, for example, the η_c could form bound states even with light nuclei. In more recent calculations the estimated charmonium binding energy in nuclear systems was found to be of the order of 10 MeV [2–5].

In the present paper we study the in-medium behaviour of the J/ψ and η_c using QCD sum rules [6]. The QCD sum rule approach connects the spectral density of a given current correlation function via a dispersion relation with the QCD operator product expansion (OPE). In-medium QCD sum rules have so far been applied only for light quark systems, in order to study possible shifts of the in-medium masses of nucleons [7–9] and vector mesons [10]. Such calculations suffer from uncertainties, e.g. due to assumptions about factorization of four-quark condensates which may not be justified. As we shall see, in-medium QCD sum rules applied to heavy quark systems are expected to be more reliable. Up to dimension four, the order to which the vacuum sum rules for hadrons involving heavy quarks are commonly expanded, all condensate parameters are quite well known and there are no ambiguities in the OPE. We also find that uncertainties caused by possibly large hadronic in-medium decay widths are much smaller than for light-quark systems.

Our starting point is the time ordered current-current correlation function of two heavy quark currents in nuclear matter,

$$\Pi(\omega, \vec{q}) = i \int d^4x \, e^{iq\cdot x} \langle |T[j(x)j(0)]| \rangle_{n.m.}. \tag{1}$$

Here $q = (\omega, \vec{q})$, and $|\rangle_{n.m.}$ is the ground state of nuclear matter which we take to be at rest. For the J/ψ we take the vector current $j^V_\mu = \bar{c}\gamma_\mu c$ and for the η_c, we use the pseudoscalar current $j^P = i\bar{c}\gamma_5 c$. In the region of large and positive $Q^2 = \vec{q}^{\,2} - \omega^2$ we can express the correlation function through an operator product expansion (short distance expansion) [11] and write the left hand side of eq.(1) as

$$\Pi(\omega, \vec{q}) = \sum_n C_n(\omega, \vec{q}) \, \langle O_n \rangle. \tag{2}$$

Here the O_n are operators of (mass) dimension n, renormalized at a scale μ^2, and C_n are the perturbative Wilson coefficients.

At baryon densities ρ_N for which the chemical potential is small compared to the scale μ separating short and long distance phenomena, all density effects can be put into the ρ_N dependence of the condensates $\langle O_{2n} \rangle$, and we can use the perturbative Wilson coefficients calculated in the vacuum [10,12]. In heavy quark systems the expansion of quark operators in terms of inverse powers of the large quark mass permits to express them entirely in terms of gluonic operators [6,13,14]. In the vacuum only the scalar gluon condensate $\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$ contributes up to dimension four. In nuclear matter, an additional contribution involving in-medium expectation values of the twist-2 tensorial gluon operator $\langle \frac{\alpha_s}{\pi} G_{\alpha\sigma} G^\sigma_\beta \rangle$ enters. We discuss this new term in some detail.

We will use the linear, low-density approximation [15] for the in-medium condensates:

$$\langle O \rangle_{n.m.} = \langle O \rangle_0 + \frac{\rho_N}{2m_N} \langle N|O|N \rangle \tag{3}$$

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†Alexander von Humboldt fellow
*work supported in part by BMBF, GSI, KOSEF and BSRI-98-2425

where $\langle\rangle_0$ represents the vacuum expectation value, and the nucleon state (taken at rest in eq. (3)) is normalized as $\langle N(p')|N(p)\rangle = 2p_0(2\pi)^3\delta^3(\vec{p}-\vec{p}')$. The in-medium changes of the condensates can then be related to the nucleon expectation values of the corresponding operators. For the traceless and symmetric gluonic twist-2 tensor operator we write

$$\langle N(p)|\frac{\alpha_s}{\pi}G^{\alpha\sigma}G^{\beta}{}_{\sigma}|N(p)\rangle = -(p^\alpha p^\beta - \frac{1}{4}g^{\alpha\beta}p^2)\frac{\alpha_s}{\pi}A_G \tag{4}$$

where m_N is the nucleon mass and A_G is related to the following moment of the gluon distribution function G:

$$A_G(\mu^2) = 2\int_0^1 dx\, x\, G(x,\mu^2). \tag{5}$$

It represents twice the momentum fraction carried by gluons in the nucleon. We take $A_G(8m_c^2) \simeq 0.9$ [16] at the scale μ used previously by Reinders et al. [13,14]. The scalar gluon condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$ changes with density according to

$$\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_{n.m.} = \langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle_0 - \frac{8}{9}m_N^0\rho_N, \tag{6}$$

where $m_N^0 \simeq 750\,\mathrm{MeV}$ is the nucleon mass in the chiral limit [17].

For the J/ψ current, using the background field technique [18], we find that the additional contribution arising from the twist-2 operator looks as follows,

$$\Delta\Pi^V_{\mu\nu}(q) = \langle\frac{\alpha_s}{\pi}G^{\alpha\sigma}G^{\beta}{}_{\sigma}\rangle\frac{1}{Q^4}\Bigg[(-g_{\mu\nu}q_\alpha q_\beta + g_{\mu\alpha}q_\nu q_\beta + q_\mu q_\alpha g_{\nu\beta} + g_{\mu\alpha}g_{\nu\beta}Q^2)\times \left(\frac{1}{2}+\left(1-\frac{Q^2}{3m_c^2}\right)J_1 - \frac{3}{2}J_2\right) + (g_{\mu\nu}-q_\mu q_\nu/q^2)q_\alpha q_\beta\left(-\frac{2}{3}+2J_1-2J_2+\frac{2}{3}J_3\right)\Bigg], \tag{7}$$

where $J_N = \int_0^1 dx[1+x(1-x)Q^2/m_c^2]^{-N}$. In the present work we study the $c\bar{c}$-system at rest (relative to the surrounding nuclear matter) and set $\vec{q}=0$, so that eqs. (1-2) refer to the Euclidean region $\omega^2 = -Q^2 < 0$. Then there is only one invariant function,

$$\tilde{\Pi}^V(-Q^2=\omega^2) = -\frac{1}{3\omega^2}g^{\mu\nu}\Pi^V_{\mu\nu}(\omega,\vec{q}=0), \tag{8}$$

which reduces to the usual vacuum polarization function when the nuclear density goes to zero.

Similarly, for the pseudoscalar case, the gluonic twist-2 correction in the OPE has the following form:

$$\Delta\Pi^P(q) = \langle\frac{\alpha_s}{\pi}G^{\alpha\sigma}G^{\beta}{}_{\sigma}\rangle\frac{q_\alpha q_\beta}{Q^4}\times \left(\frac{1}{2}+\frac{1}{3}\left(1-\frac{Q^2}{m_c^2}\right)J_1 - \frac{1}{6}J_2 - \frac{2}{3}J_3\right) \tag{9}$$

Here we introduce the (dimensionless) polarization function

$$\tilde{\Pi}^P(-Q^2=\omega^2) = \frac{\Pi^P(\omega,\vec{q}=0)}{\omega^2}, \tag{10}$$

which reduces in the limit $\rho_N \to 0$ to the usual vacuum polarization function.

Our analysis is based on the moments of the polarization function $\tilde{\Pi}^J$ with $J=V,P$ referring to the vector or pseudoscalar channels. The n-th moment is connected, on the other side, with a dispersion integral involving $\mathrm{Im}\tilde{\Pi}^J$,

$$M_n^J \equiv \frac{1}{n!}\left(\frac{d}{d\omega^2}\right)^n\tilde{\Pi}^J(\omega^2)\Bigg|_{\omega^2=-Q_0^2} = \frac{1}{\pi}\int_{4m_c^2}^\infty\frac{\mathrm{Im}\tilde{\Pi}^J(s)}{(s+Q_0^2)^{n+1}}ds, \tag{11}$$

at a fixed $Q_0^2 = 4m_c^2\xi$. Direct evaluation of these moments using the OPE gives

$$M_n^J(\xi) = A_n^J(\xi)\left[1 + a_n^J(\xi)\alpha_s + b_n^J(\xi)\phi_b + c_n^J(\xi)\phi_c\right]. \tag{12}$$

The common factor A_n^J results from the bare loop diagram. The coefficient a_n^J takes into account perturbative radiative corrections, while b_n^J is associated with the gluon condensate term,

$$\phi_b = \frac{4\pi^2}{9}\frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{(4m_c^2)^2}, \tag{13}$$

The coefficients A_n^J, a_n^J and b_n^J are listed in ref. [13]. The new contribution from the twist-2 gluon operator involves

$$\phi_c = -\frac{2\pi^2}{3}\frac{\frac{\alpha_s}{\pi}A_G}{(4m_c^2)^2}m_N\rho_N. \tag{14}$$

For the additional Wilson coefficient c_n we find in the vector channel:

$$c_n^V(\xi) = b_n^V(\xi) - \frac{4n(n+1)}{3(2n+5)(1+\xi)^2}\frac{F(n+2,\frac{3}{2};n+\frac{7}{2};\frac{\xi}{1+\xi})}{F(n,\frac{1}{2};n+\frac{5}{2};\frac{\xi}{1+\xi})} \tag{15}$$

and in the pseudoscalar channel we obtain

$$c_n^P(\xi) = b_n^P(\xi) - \frac{4n(n+1)}{(1+\xi)} \frac{F(n+1, -\frac{1}{2}; n+\frac{3}{2}; \frac{\xi}{1+\xi})}{F(n, \frac{1}{2}; n+\frac{3}{2}; \frac{\xi}{1+\xi})} \tag{16}$$

with hypergeometric functions $F(a, b; c; z)$. By comparison with ref. [13] we see that the c_n's differ very little from the b_n's. From the resulting term $b_n^J(\phi_b + \phi_c)$ in eq. (12) one then observes that the gluon condensate effectively changes by the following density dependent correction:

$$\begin{aligned}\langle\frac{\alpha_s}{\pi}G^2\rangle_0 &\to \langle\frac{\alpha_s}{\pi}G^2\rangle_0 - \left(\frac{8}{9}m_N^0 + \frac{3}{2}m_N\frac{\alpha_s}{\pi}A_G\right)\rho_N \\ &\simeq \langle\frac{\alpha_s}{\pi}G^2\rangle_0(1 - 0.06\rho_N/\rho_0),\end{aligned} \tag{17}$$

using $\langle\frac{\alpha_s}{\pi}G^2\rangle_0 = (0.35\text{GeV})^4$ and $\rho_0 = 0.17\,\text{fm}^{-3}$.

The spectral function under the integral on the r.h.s. of eq.(11) is parameterized as

$$\text{Im}\tilde{\Pi}(s) = \sum_i f_i \delta(s - m_i^2) + c\,\theta(s - s_0)\left(1 + \frac{\alpha_s(s)}{4\pi}\right), \tag{18}$$

in terms of a sum over low-lying resonances and a continuum part starting from s_0, with $c = 1/4\pi^2$ in the vector channel and $c = 3/8\pi^2$ in the pseudoscalar channel. In the vector channel the couplings f_i of the $\bar{c}\gamma_\mu c$ current to the J/ψ, ψ', ψ'', ... resonances are determined by their measured decay widths into e^+e^-. Inserting eq. (18) into eq. (11), it is convenient to write

$$M_n^J(\xi) = \frac{f_0}{\pi(m^2 + Q_0^2)^{n+1}}[1 + \delta_n^J(\xi)], \tag{19}$$

where m is the mass of the lowest state, the one of interest. The contributions of higher resonances as well as the continuum are absorbed in δ_n^J. Clearly, the relative importance of these higher energy parts of the spectrum decreases with increasing n. It is common practice to take the ratio of two neighboring moments, $M_{n-1}/M_n = (m^2 + Q_0^2)(1 + \delta_{n-1})/(1 + \delta_n)$, so that f_0 drops out and one can focus on the mass m. For $n \geq 5$ it turns out that $(1+\delta_{n-1})/(1+\delta_n)$ is close to one. Then the moment ratio does not depend on details of the higher resonances and continuum parts of the spectrum, and we have

$$m^2 \simeq \frac{M_{n-1}(\xi)}{M_n(\xi)} - 4m_c^2\xi. \tag{20}$$

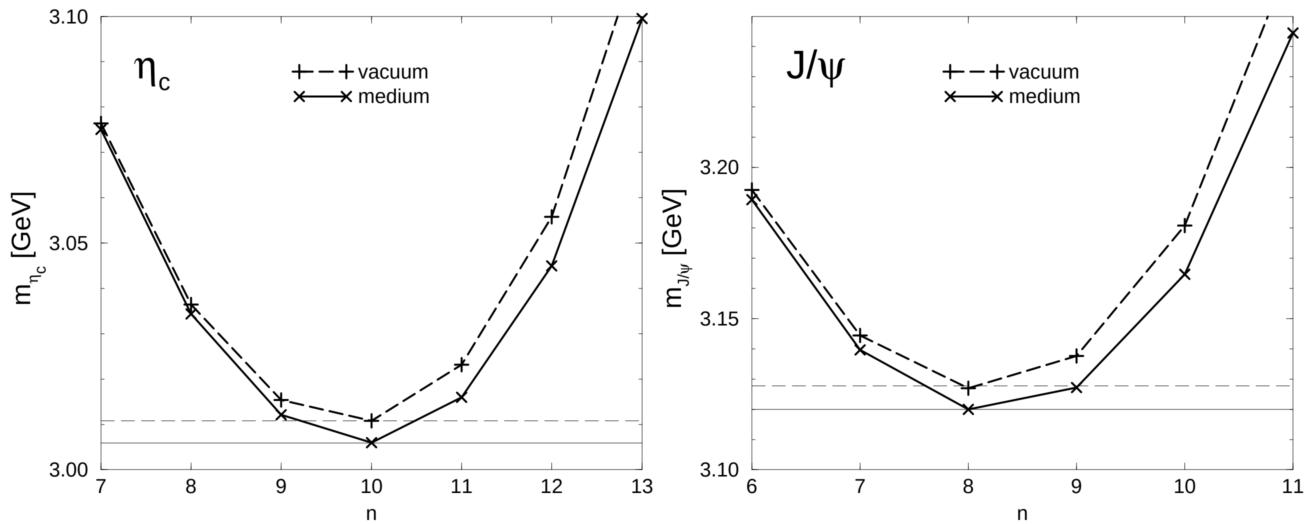

FIG. 1. The η_c and J/ψ masses calculated according to eq. (20) for different n at $\xi = 1$. We show the result in medium at $\rho_N = 0.17\,\text{fm}^{-3}$ (solid line) in comparison with the vacuum result (dashed line).

The actual mass determination is done using moments in the range $7 \leq n \leq 11$ and choosing $\xi = 1$, just as in the vacuum case studied previously [13,14]. This range minimizes the sensitivity to details of the high-energy spectrum. Going to larger n would not be justified without introducing additional, unknown condensates of higher dimension in the OPE.

In Fig. 1 we show the results for in-medium masses (solid lines) of the J/ψ and η_c at normal nuclear matter density ($\rho_N = \rho_0 = 0.17\,\text{fm}^{-3}$) in comparison with their vacuum values (dashed lines). Using $\alpha_s(8m_c^2) = 0.21$, $m_c = 1.24\,\text{GeV}$, $\phi_b = 1.8 \cdot 10^{-3}$ in the vacuum and $\phi_b = 1.7 \cdot 10^{-3}$, $\phi_c = -1.25 \cdot 10^{-5}$ in nuclear matter, we find the following mass shifts taken at the minimal values of eq. (20):

$$\Delta m_\psi \simeq -7\,\text{MeV}, \tag{21}$$

$$\Delta m_{\eta_c} \simeq -5\,\text{MeV}. \tag{22}$$

These shifts depend only very weakly on our choice of parameters. Taking higher resonances explicitly into ac-

count would lead to somewhat smaller mass shifts.

The sensitivity to possible enlarged in-medium widths of the J/ψ or η_c turns out to be marginal. In fact no inelastic channels exist for ground state $c\bar{c}$ mesons at rest interacting with a nucleon. Even if such channels were present, they would not affect the mass shift analysis unless the corresponding widths would reach magnitudes of 100 MeV or larger. This is in qualitative contrast to light quark systems, such as the ρ meson, for which the in-medium width becomes so large that the QCD sum rule analysis of a possible mass shift becomes ambiguous and inconclusive [19,20].

In summary our in-medium QCD sum rule analysis, with the operator product expansion calculated up to dimension four, predicts attractive mass shifts of about 5-10 MeV for J/ψ and η_c in nuclear matter. This corresponds to small J/ψ- and η_c-nucleon scattering lengths $a = -\mu_r \Delta m/(2\pi\rho_N) \simeq (0.1 - 0.2)\,\mathrm{fm}$ (μ_r is the meson-nucleon reduced mass). Our results for the mass shifts of the lowest $\bar{c}c$ states are surprisingly close to those reported in ref. [3–5]. Most of the calculated mass shift comes from the density dependence of the gluon condensate. The new term related to the fraction of momentum carried by gluons in the nucleon contributes less than 10% to the total effect. The influence of the decay widths is expected to be very small, at least for J/ψ and η_c at rest [4]. Of course, for charmonium systems traversing nuclear matter at high energy, the scattering amplitudes can develop substantial imaginary parts from reactions with nucleons producing open charm [21].

After submission of this paper a similar calculation has been reported in ref. [22]

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